

MST125

TMA 01

2018J

Covers Units 2, 3, 4 and 5

Cut-off date 12 December 2018

You can submit this TMA either by post to your tutor or electronically as a PDF file by using the University's online TMA/EMA service.

Before starting work on it, please read the document *Student guidance for preparing and submitting TMAs*, available from the 'Assessment' area of the MST125 website.

The work that you submit should include your working as well as your final answers. Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not include the use of any other mathematical software. If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

Your work should be written in good mathematical style, as demonstrated by the example and activity solutions in the study units. You should explain your solutions carefully, using appropriate notation and terminology, and write in sentences. As usual, you should simplify algebraic answers where possible. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 7. You do not have to submit any work for Question 7.

Question 1 – 20 marks

You should be able to answer this question after studying Unit 2.

The purpose of this question is to allow you to demonstrate your mathematical typesetting skills. Therefore you *must* typeset your work for parts (a) and (b) of this question using one of $\text{\LaTeX}$, Word (version 2007 or later), LibreOffice or OpenOffice.

If you are submitting your TMA by post, then please *include a printout* of your response to Question 1 in your TMA.

If you are using the online TMA/EMA service, but not typesetting the rest of the TMA, then please do *one* of the following.

- (i) Print out your response to Question 1, scan it with the rest of your TMA, and save the scan as a single PDF file. Then submit the PDF file using either the ‘Submit’ button on the Assessment page of the MST125 website or the link to the online TMA/EMA service from your StudentHome page.
- (ii) Combine the PDF for Question 1 with the PDF of the rest of the TMA into a single PDF file. Then submit the combined PDF file using either the ‘Submit’ button on the Assessment page of the MST125 website or the link to the online TMA/EMA service from your StudentHome page.

Guidance on submitting PDF files using the online TMA/EMA service is provided in the document *Student guidance for preparing and submitting TMAs* available from the ‘Assessment’ area of the MST125 website.

- (a) On the first line, type the heading ‘MST125 TMA 01 Question 1’, making it centred and bold.

On the second line, type and centre your name and personal identifier.

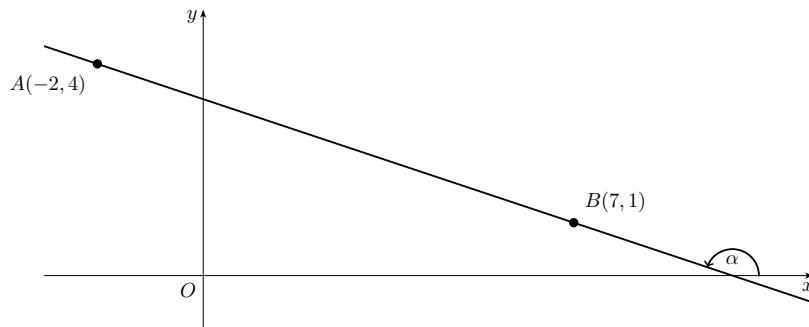
On the third line, type the name of the typesetting software that you are using.

[2]

(b) A student is working on the following problem.

(Note that you are not asked to provide a solution to this problem.)

The diagram below shows the straight line through the points $A(-2, 4)$ and $B(7, 1)$. Let α be the obtuse angle that this line makes with the positive direction of the x -axis, as shown. The scales on the two axes are equal.



- (i) Calculate the distance AB .
- (ii) Calculate the gradient of the line through A and B .
- (iii) Find the angle α in radians to two decimal places.

The student has produced the draft solution shown below.

(i) The distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Hence the distance between $A(-2, 4)$ and $B(7, 1)$ is

$$\begin{aligned} AB &= \sqrt{(7 - (-2))^2 + (1 - 4)^2} \\ &= \sqrt{9^2 + (-3)^2} \\ &= 3\sqrt{10}. \end{aligned}$$

(ii) The gradient m of the line through (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Hence, the gradient of the line through $(-2, 4)$ and $(7, 1)$ is

$$m = \frac{1 - 4}{7 - (-2)} = \frac{-3}{9} = -\frac{1}{3}.$$

(iii) The gradient of the line is $-\frac{1}{3}$. Hence $\tan \alpha = -\frac{1}{3}$.

Let ϕ be the acute angle that the line makes with the negative direction of the x -axis. Then

$$\tan \phi = \frac{1}{3}.$$

$$\text{So } \phi = \tan^{-1}\left(\frac{1}{3}\right) = 0.321\dots$$

Hence

$$\alpha = \pi - 0.321\dots = 2.819\dots$$

Therefore the angle α is 2.82 radians (to 2 d.p.).

Typeset this draft solution, using the same content and a similar layout.

Take care to centre or indent each equation that is displayed on a line by itself, and to align equals signs where appropriate.

Marks will be awarded for

- | | |
|---|-----|
| (i) layout and accuracy | [9] |
| (ii) mathematical notation. | [7] |
| (c) Do you intend to typeset your TMA solutions? Justify your decision briefly. | [2] |

Question 2 – 5 marks

You should be able to answer this question after studying Unit 3.

- (a) Use a divisibility test to determine whether the number 386 386 386 386 is divisible by 9. [2]
- (b) Using Fermat's little theorem, find the least residue of 5^{75} modulo 37. [3]

Question 3 – 20 marks

You should be able to answer this question after studying Unit 3.

- (a) (i) Use Euclid's algorithm to find a multiplicative inverse of 23 modulo 66, and hence solve the linear congruence
- $$23x \equiv 7 \pmod{66}. \quad [8]$$
- (ii) Explain why the following linear congruence has no solutions:
- $$3x \equiv 13 \pmod{66}. \quad [2]$$
- (iii) Solve the linear congruence
- $$55x \equiv 22 \pmod{66}. \quad [5]$$
- (b) The affine cipher E is given by
- $$E(x) \equiv 15x - 2 \pmod{26}.$$

The conversion table for letters and numbers is shown below.

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

- (i) Show that 7 is a multiplicative inverse of 15 modulo 26. [1]
- (ii) Suppose that you receive the enciphered message
- $$23, 0, 15.$$

Decipher the message and deduce the word that was sent. [4]

Question 4 – 15 marks

You should be able to answer this question after studying Unit 4.

(a) Consider the equation

$$9x^2 - 25y^2 - 4 = 0.$$

- (i) Show that this equation represents a conic in standard position, and identify the type of conic. [2]
- (ii) Find all the vertices and asymptotes of the conic, if there are any. [2]
- (iii) Sketch the conic, labelling all vertices with their coordinates and any asymptotes with their equations. [2]
- (iv) Find the eccentricity, foci and directrices of the conic. [3]
- (v) Write down a parametrisation of the part of this conic that lies in the second quadrant (not including any points that lie on the x -axis or y -axis). Make sure that you include any restrictions that are needed on the values of the parameter. [4]

(b) Consider the conic with equation

$$3x^2 + 14y^2 = 10.$$

Plot the conic using Maxima, and provide a printout or screenshot of your plot (depending on how you have chosen to prepare your TMA). [2]

Question 5 – 10 marks

You should be able to answer this question after studying Unit 4.

Two objects, A and B , are moving along two different straight lines at constant speeds. With reference to a particular coordinate system in which distance is measured in metres, the position of A at time t (in minutes) is $(4 - 2t, 7 - t)$, and the position of B is $(1 + t, 3 + 2t)$.

(a) Find the equation of the line that A moves along. [2]

(b) Let d be the distance between A and B at time t . Show that an expression for d^2 in terms of t is given by

$$d^2 = 18t^2 - 42t + 25. \quad [3]$$

(c) A student attempts to find the shortest distance between A and B as shown below. There are two lines where the working does not follow on from the previous line.

$$\begin{aligned} d^2 &= 18t^2 - 42t + 25 \\ &= 18 \left(t^2 - \frac{7}{3}t \right) + 25 \\ &= 18 \left(t - \frac{7}{6} \right)^2 - \left(\frac{7}{6} \right)^2 + 25 \\ &= 18 \left(t - \frac{7}{6} \right)^2 + 23.6 \dots \end{aligned}$$

The minimum value of d^2 occurs when $t = \frac{7}{6}$.

Hence the minimum distance is 23.6 m (to 1 d.p.).

Find and describe the two mistakes. Write out a correct solution. [5]

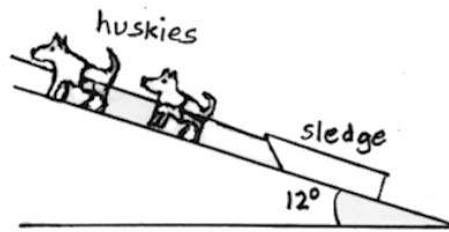
Question 6 – 25 marks

You should be able to answer this question after studying Unit 5.

An Arctic expedition has a team of husky dogs which are attached to a sledge by a rope. The dogs pull the sledge across the snow. At night, the food tub is suspended from two tree branches by ropes to keep it out of reach of animals.

Model the sledge and the food tub as particles and the ropes as model strings. Take the magnitude of the acceleration due to gravity to be $g = 9.8 \text{ ms}^{-2}$, and round your answers to two significant figures.

- (a) Part way up a slope, the team stops for a rest. The huskies exert a pulling force on the sledge which acts in a direction that is parallel to and up the slope. The sledge is on the point of slipping up the slope. The angle of inclination of the slope is 12° and the coefficient of static friction between the snow and the sledge is 0.15. The laden sledge has a mass of 200 kg.



Model the slope as a plane.

- (i) Draw a force diagram showing the forces acting on the sledge. Define the symbols that you use to denote the forces. Choose the unit vector $\mathbf{i}$ to be parallel to and pointing down the slope and the unit vector $\mathbf{j}$ to be perpendicular to the slope and pointing upwards. Mark these vectors on your diagram. [5]
- (ii) Express the forces in component form, in terms of unknown magnitudes where appropriate. Hence determine the magnitude (in newtons) of the tension in the rope. [9]
- (b) (i) The food tub has a mass of 20 kg. The left-hand rope makes an angle of 35° and the right-hand rope makes an angle of 55° with the upward vertical. Draw a force diagram showing the forces acting on the food tub. Define the symbols that you use to denote the forces. Choose the unit vector $\mathbf{i}$ to point horizontally to the right and the unit vector $\mathbf{j}$ to point vertically upwards. Mark these vectors on your diagram. [3]
- (ii) Express the forces in component form, in terms of unknown magnitudes where appropriate. Hence determine the magnitude (in newtons) of the tension in the left-hand rope. [8]

Question 7 – 5 marks

Five marks on this assignment are allocated for good mathematical communication in Questions 2 to 6.

You do not have to submit any extra work for Question 7, but you are advised to check through your assignment carefully, making sure that you have explained your working clearly, used notation correctly, written in sentences and rounded answers as requested. [5]